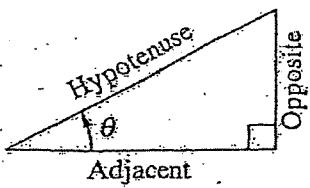


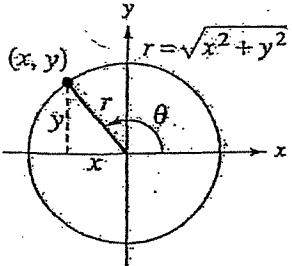
## Definition of the Six Trigonometric Functions

Right triangle definitions, where  $0 < \theta < \pi/2$ .



$$\begin{aligned}\sin \theta &= \frac{\text{opp.}}{\text{hyp.}} & \csc \theta &= \frac{\text{hyp.}}{\text{opp.}} \\ \cos \theta &= \frac{\text{adj.}}{\text{hyp.}} & \sec \theta &= \frac{\text{hyp.}}{\text{adj.}} \\ \tan \theta &= \frac{\text{opp.}}{\text{adj.}} & \cot \theta &= \frac{\text{adj.}}{\text{opp.}}\end{aligned}$$

Circular function definitions, where  $\theta$  is any angle.



$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

## Reciprocal Identities

$$\begin{aligned}\sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} & \tan u &= \frac{1}{\cot u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u} & \cot u &= \frac{1}{\tan u}\end{aligned}$$

## Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

## Pythagorean Identities

$$\begin{aligned}\sin^2 u + \cos^2 u &= 1 \\ 1 + \tan^2 u &= \sec^2 u \\ 1 + \cot^2 u &= \csc^2 u\end{aligned}$$

## Cofunction Identities

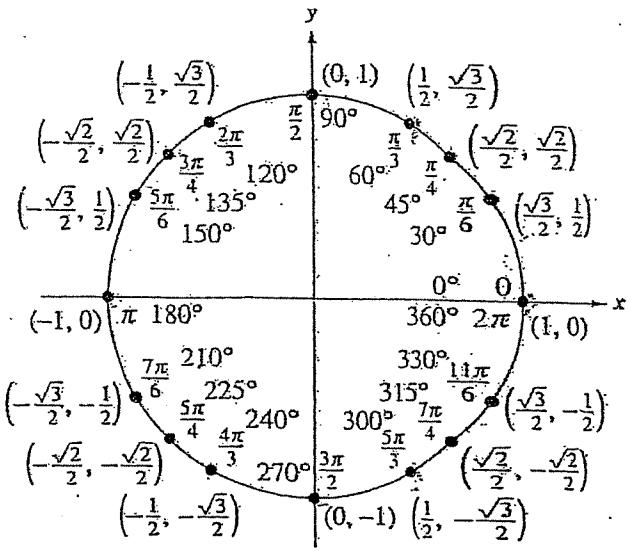
$$\begin{aligned}\sin\left(\frac{\pi}{2} - u\right) &= \cos u & \cot\left(\frac{\pi}{2} - u\right) &= \tan u \\ \cos\left(\frac{\pi}{2} - u\right) &= \sin u & \sec\left(\frac{\pi}{2} - u\right) &= \csc u \\ \tan\left(\frac{\pi}{2} - u\right) &= \cot u & \csc\left(\frac{\pi}{2} - u\right) &= \sec u\end{aligned}$$

## Even/Odd Identities

$$\begin{aligned}\sin(-u) &= -\sin u & \cot(-u) &= -\cot u \\ \cos(-u) &= \cos u & \sec(-u) &= \sec u \\ \tan(-u) &= -\tan u & \csc(-u) &= -\csc u\end{aligned}$$

## Sum and Difference Formulas

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$



## Double-Angle Formulas

$$\begin{aligned}\sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

## Power-Reducing Formulas

$$\begin{aligned}\sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u}\end{aligned}$$

## Sum-to-Product Formulas

$$\begin{aligned}\sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)\end{aligned}$$

## Product-to-Sum Formulas

$$\begin{aligned}\sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\cos(u+v) - \sin(u-v)]\end{aligned}$$